The Stochastic Axion Scenario

Adam Scherlis
with Peter Graham
[1805.07362] to appear in PRD

July 26, 2018

See talk by Takahashi

Introduction: Axion Dark Matter

Two scenarios for the axion:

- Post-Inflationary: PQ breaks after inflation
  Precise mass needed to get axion DM
- Pre-Inflationary: PQ breaks before/during inflation
  Range of compatible masses

Usual lore: overclosure bound (or tuning/anthropics) at low mass

- Or new cosmology/axion models [Agrawal,Marques-Tavares,Xue; Nomura,Rajendran,Sanches; Dine,Fischler; Steinhardt,Turner; Lazarides,Schaefer,Seckel,Shafi; Kawasaki,Moroi,Yanagida; Dvali; Choi,Kim,Kim; Banks,Dine; Banks,Dine,Graesser]

Usual picture:

All masses are natural w/o new physics

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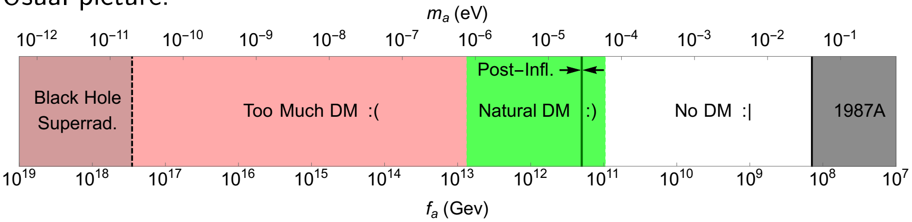

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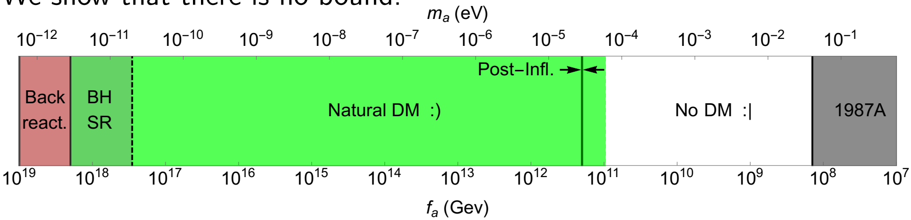
We show that there is no bound:



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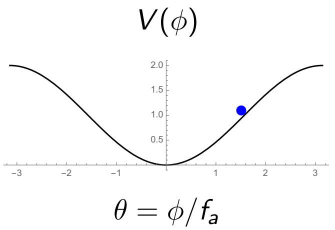
Introduction: Misalignment Mechanism

After PQ breaking: axion "frozen" at $f_a \theta_0$

- Final abundance: depends on f_a, θ_0
- Fix DM abundance: relation $f_a \leftrightarrow \theta_0$



Where does θ_0 come from?

- Post-inflation PQ breaking: Temperature reaches f_a
  - Averaged θ_0, string decay $\Longrightarrow$ θ_C
  - Single f_a for axion DM ("Classical Window")
- Pre-inflation PQ breaking: Temperature stays below f_a
  - $\theta_0 = \mathcal{O}(1)$ implies $f_a = \mathcal{O}(10^{12}\ \text{GeV})$ ("Natural")
  - Higher f_a requires smaller θ_0

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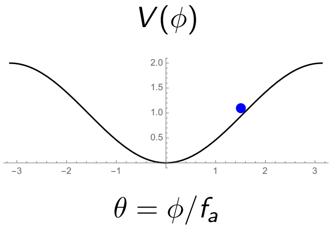

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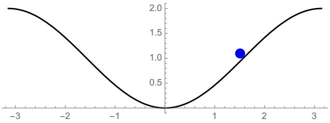
Introduction: Misalignment Mechanism

$V(\phi)$



$\theta = \phi / f_a$

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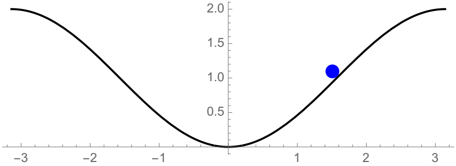
Mechanics: Hopping and Sliding

Where does θ_0 *really* come from?
Scalar field dynamics during inflation:
Every e-fold, two things happen:



- Sliding: classical slow-roll towards minimum

$$\phi \mapsto \phi - \frac{m^2}{3H^2}\phi$$

- Hopping: quantum fluctuations ("random walk")

$$\phi \mapsto \phi \pm \frac{H}{2\pi}$$

Eventually reaches equilibrium (independent of initial conditions and N)

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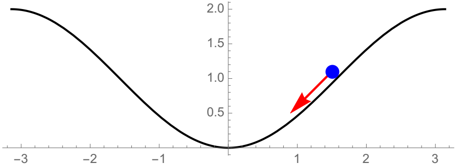

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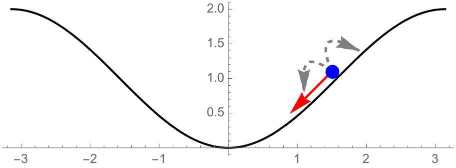

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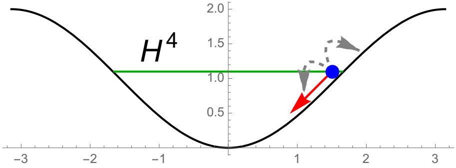

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Mechanics: Example Animation

See PPT version for animation!

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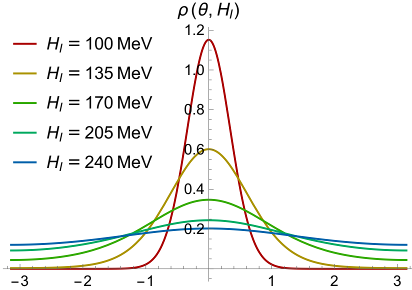
Mechanics: Equilibrium Distribution

Fokker-Planck equation: $\dot{\rho}(\phi, t) = \frac{1}{3H_I}\partial_\phi(V'(\phi)\rho(\phi, t)) + \frac{H_I^3}{8\pi^2}\partial^2_{\phi\phi}\rho(\phi, t)$

Distribution of θ over many patches:

- $m^2 \langle \phi^2 \rangle \approx H_I^4$
- Naturally small but nonzero
- θ very uniform for individual patch

- $H_I < 800$ MeV: axion has a mass
  - $H_I < 200$ MeV: distribution is Gaussian
  - $H_I > 200$ MeV: distribution is flat



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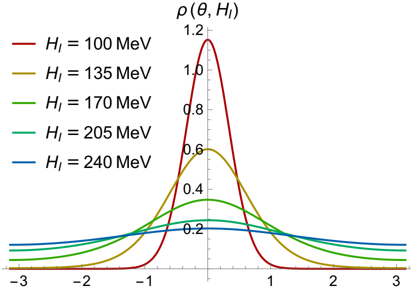

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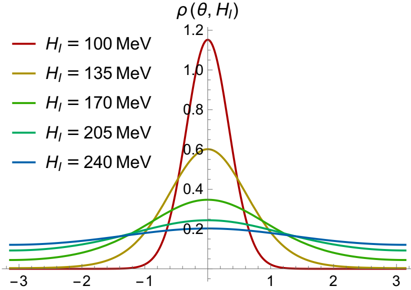

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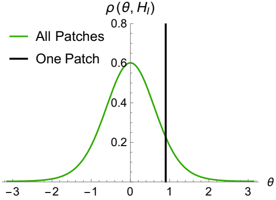
Isocurvature

- Random $\mathcal{O}(H_I)$ hops build up over $> 10^{20}$ e-folds
- Inhomogeneities stretch and leave horizon
- Hops from last ~ 60 efolds remain inhomogeneous



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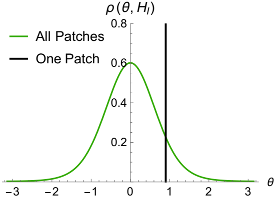

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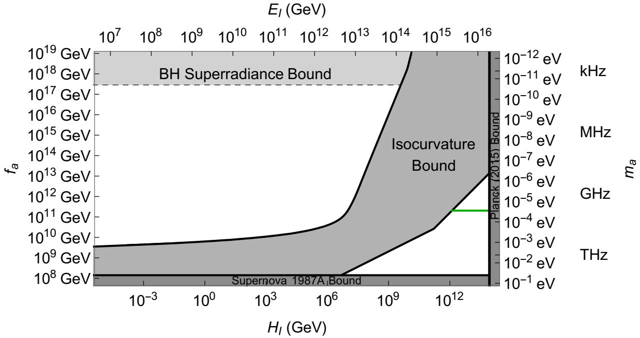
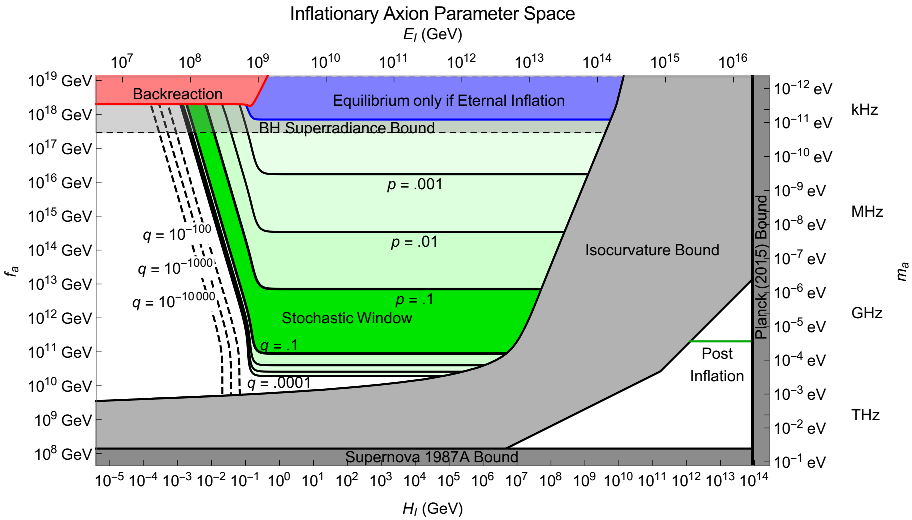


For N large enough to reach equilibrium:

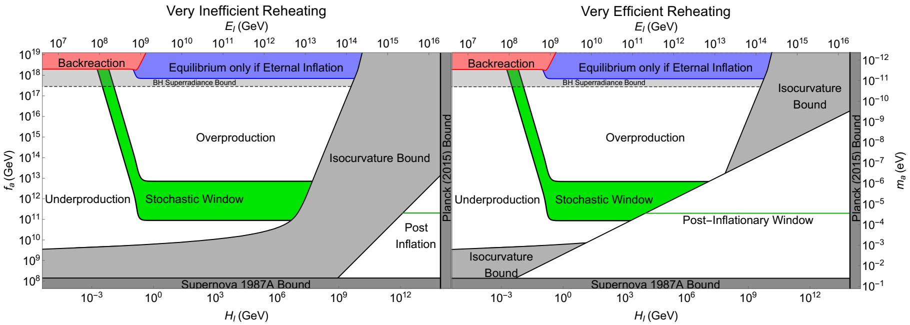


Results: Reheating

All of this is irrelevant if temperature hits f_a during

1. Inflation: $T_{dS} \sim H_I$
2. Reheating: $T_{rh} \sim \epsilon_{eff} \sqrt{m_P H_I}$

Inefficient reheating: $T_{dS} > T_{rh}$ at high H_I



Note: $T_{rh} \gg$ TeV unless extremely inefficient.

Results: Inflation

Main caveat: need LOTS of inflation

- Low mass: $H_I \lesssim \Lambda_{QCD} \longleftrightarrow E_I \lesssim 10^9$ GeV
- Value of H_I determines width of θ_0 distribution
- Relaxation time: $t_{rel} = 3\frac{H_I}{m_a^2}$ or $N_{rel} = 3\frac{H_I^2}{m_a^2}$

Some points that naturally have the right abundance:

m_a	f_a	H_I	t_{rel}	N_{rel}
14 kHz	10^{17} GeV	10 MeV	200,000 yr	10^{35}
14 MHz	10^{14} GeV	100 MeV	2 years	10^{31}
14 GHz	10^{11} GeV	1000 MeV	0.5 seconds	10^{24}

- **See also: Guth-Takahashi-Yin [1805.08763]**
  (includes low-H_I hilltop potential)

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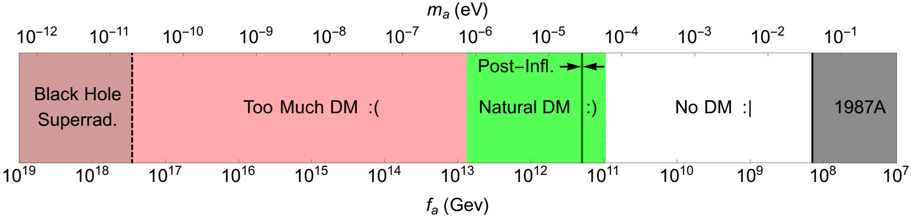
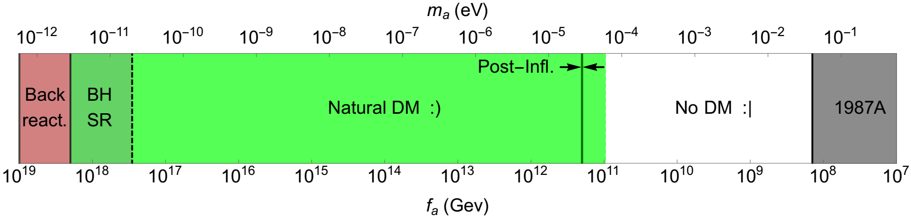
Conclusion

Conventional claim:



But actually, unless we make assumptions about inflation:



So it's important to search the entire mass range experimentally.

Backup: Fokker-Planck Formalism

Fokker-Planck equation: $\dot{\rho}(\phi, t) = \frac{1}{3H_I}\partial_\phi(V'(\phi)\rho(\phi, t)) + \frac{H_I^3}{8\pi^2}\partial^2_{\phi\phi}\rho(\phi, t)$
Change variables,

$$\rho(\phi, t) = \psi_0(\phi)\psi(\phi, t)$$

$$\psi_0(\phi) := \exp(-\nu(\phi)) = \exp\left(-\frac{4\pi^2}{3H_I^4}V(\phi)\right)$$

to get Schroedinger-like equation:

$$-\frac{4\pi^2}{H_I^3}\dot{\psi}(\phi, t) = -\frac{1}{2}\psi''(\phi, t) + \frac{1}{2}\left[-\nu''(\phi) + \nu'(\phi)^2 - \frac{3}{M_P^2}\nu(\phi)\right]\psi(\phi, t)$$

Eigenfunctions $\rho_i = \psi_0\psi_i$ are quasinormal modes
$\rho_0 = \psi_0^2$ is equilibrium distribution
Smallest positive eigenvalue is relaxation rate

Backup: Backreaction

- Expansion rate is related to energy,

$$3H^2 m_P^2 = V$$

- Axion contributes a small amount,

$$V = V_I + V_a$$

$$V_I \gg V_a$$

- Regions with large θ expand (slightly) faster

This effect suddenly becomes dominant for $f_a \gtrsim m_P$ at $H_I \lesssim \Lambda_{QCD}$
Nearly all patches overproduce with $\theta \to \pi$ (for some choice of measure)

Backup: Eternal Inflation

- Inflaton also has sliding and hopping
- If potential is too flat, hopping dominates
  $\Longrightarrow$ inflation becomes chaotic (eternal)
- Equivalent to minimum "speed" of inflaton, or maximum length of inflation

$$N \lesssim \frac{m_P^2}{H_I^2}$$

Relaxation time violates this bound for $f_a \gtrsim m_P$ at $H_I \gtrsim \Lambda_{QCD}$
Our analysis still works but eternal inflation introduces measure issues